Reading Assignment Debrief

- Discuss your answer to Task 1 with your group.
- Are there any questions from Sections 9.1.1 – 9.1.5 that your group wants to address?

Questions to Address:

- Level curves of $f(x,y) = \sqrt{x^2+y^2}$. ✓
- When are 2 vectors equal ✓
- Ed finity Q?
- Contour maps? ✓

Section 9.1.4 Traces

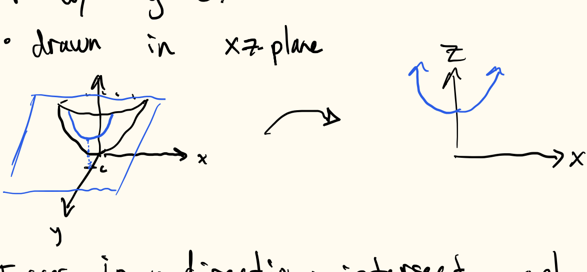
Definition 9.1.12 A trace of a function $f(x,y)$ in the x-direction is a curve defined by the equation

$$z = f(x,c).$$

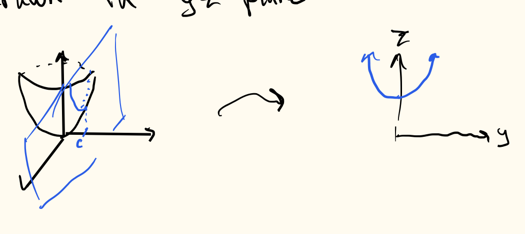
for some constant $c \in \mathbb{R}$. Similarly, a trace of f in the y-direction is a curve of the form

$$z = f(c,y)$$

Traces in x-direction: • intersect graph of f w/ $y = c$.
• drawn in xz plane



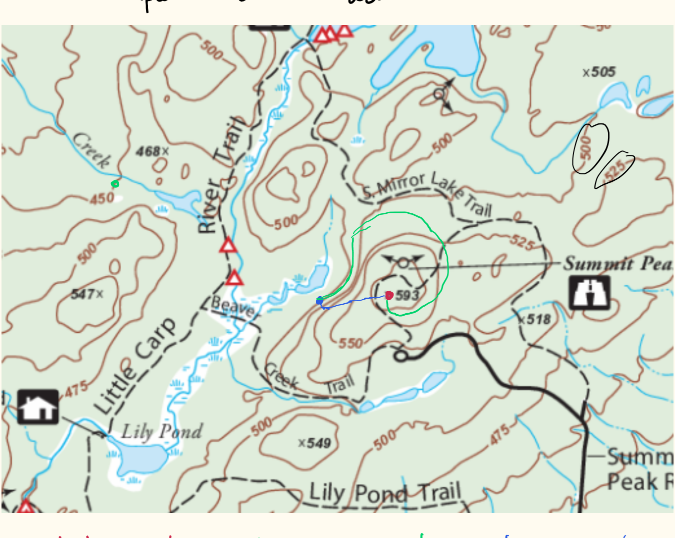
Traces in y-direction: intersect graph of f w/ $x = c$
• drawn in yz-plane



Section 9.1.5 Contour Maps and Level Curves

Activity 9.1.7

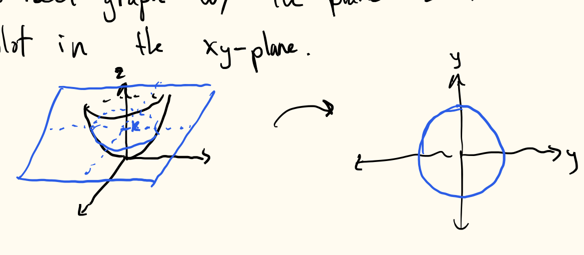
- Complete as a class.



• High point • Lowest point • Chosen point
• less steep path • Steep path

Definition 9.1.15 A level curve or level contour of a function $f(x,y)$ is a curve of the form $K = f(x,y)$ for some constant $K \in \mathbb{R}$.

level curves: • Similar to traces, but no "direction"
• intersect graph w/ the plane $z = k$
• plot in the xy-plane.

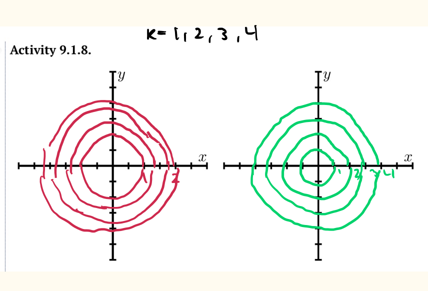


Activity 9.1.8

- Complete Activity 9.1.8 (a) and (c) (See Reading 1.2 for part (b)) w/ your group.
- Don't graph the function. We will look at the graph after.
- Class discussion.

$f(x,y) = x^2+y^2$

$k = x^2+y^2$
circles w/ $r = \sqrt{k}$

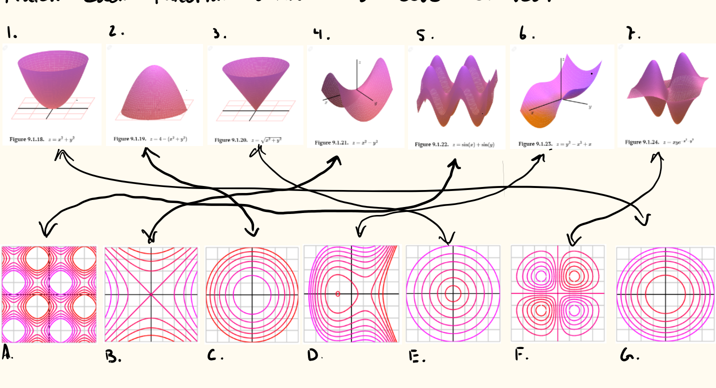


$g(x,y) = \sqrt{x^2+y^2}$
$k = \sqrt{x^2+y^2}$
$k^2 = x^2+y^2$
circles with $r = k$

(a) Looks like a bowl, see above.
(b) a cone
(c) The level curves for g and f are both concentric circles. However, the radii for g grow linearly as a function of k. The radii for f do not.

Section 9.1.6 A Gallery of Functions

Match each function with it's level curves:



End of Section 9.1.

Section 9.2 Vectors

Reading Assignment Debrief

- Discuss your answer to Tasks 5–6 w/ your group.
- Are there any questions from Sections 9.2.1 – 9.2.2 that your group wants to address?

Questions:

- When are 2 vectors equal?

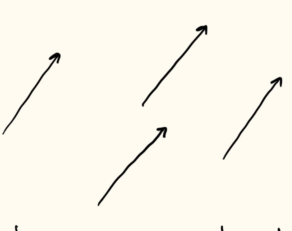
Definition 9.2.1 A vector is a quantity that possesses the attributes of both magnitude and direction.

Examples: • velocity • displacement
• force • acceleration

Non-examples: • speed • distance
• mass • rate of change of speed
• time

Geometric Representation of Vectors

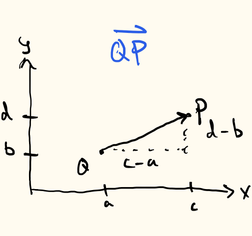
Arrows!



All these vectors are equal: they have same magnitude and direction.

Displacement Vectors Let $P = (c,d)$, $Q = (a,b)$

$\overrightarrow{QP} = \langle c-a, d-b \rangle$ Displacement Vector from Q to P.

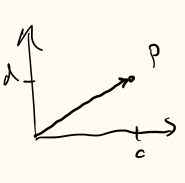


You can do the same thing in 3 or more dim.

Position Vectors Any point $P = (c,d)$ determines a unique vector by taking the initial point the origin $O = (0,0)$:

$$\overrightarrow{OP} = \langle c,d \rangle$$

This is the Position Vector for P



Activity 9.2.2

- Complete parts (d), (e), (f)
- Class discussion.

(d) $\overrightarrow{AB} = \langle -9, 7, 4.5 \rangle$
displ. between students A and projector B
(e) $\overrightarrow{AC} = \langle -11, 26, -2.5 \rangle$
displ. from student A to teacher
(f) $\overrightarrow{BC} = \langle -2, 19, -7 \rangle$
displ. from projector to teacher.

Section 9.2.3 Operations on Vectors

Activity 9.2.3

- Complete the activity and discuss w/ group.
- Class discussion.

(a) $u+v = \langle 2,3 \rangle + \langle -1,4 \rangle = \langle 1,7 \rangle$

(b) Definition: For all $a, b \in \mathbb{R}^2$,
$a+b = \langle a_1, a_2 \rangle + \langle b_1, b_2 \rangle = \langle a_1+b_1, a_2+b_2 \rangle$

(c) Definition: For all $a, b \in \mathbb{R}^3$,
$a+b = \langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle = \langle a_1+b_1, a_2+b_2, a_3+b_3 \rangle$

vector addition

(d) $\frac{1}{2}v = \langle \frac{1}{2}(-1), \frac{1}{2}4 \rangle = \langle -\frac{1}{2}, 2 \rangle$

(e) Definition: For all $c \in \mathbb{R}$, $a \in \mathbb{R}^3$,
$c \cdot a = c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$ Scalar mult.

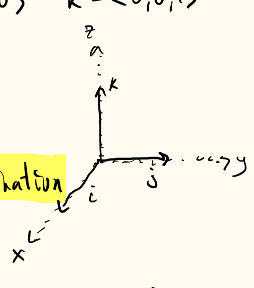
Vector Subtraction Let $u, v \in \mathbb{R}^n$. Then

$$v - u = v + (-1)u$$

Standard Unit Vectors Three special vectors in $\mathbb{R}^3$

$i = \langle 1,0,0 \rangle$ $j = \langle 0,1,0 \rangle$ $k = \langle 0,0,1 \rangle$

• They all have length 1
• Every vector $\langle a,b,c \rangle$ can be written as a linear combination of i, j, k:



$$\langle a,b,c \rangle = \langle a,0,0 \rangle + \langle 0,b,0 \rangle + \langle 0,0,c \rangle = a\langle 1,0,0 \rangle + b\langle 0,1,0 \rangle + c\langle 0,0,1 \rangle = ai + bj + ck$$

Section 9.2.4 Properties of Vector Operations

Vector addition and scalar mult. have properties similar to addition and mult of number.

Ex $u+v = \langle a,b \rangle + \langle c,d \rangle = \langle a+c, b+d \rangle = \langle c+a, d+b \rangle = \langle c,d \rangle + \langle a,b \rangle = v+u.$

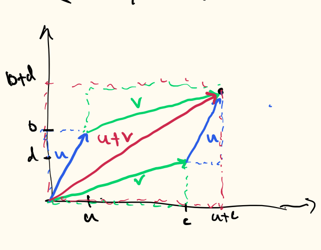
Properties of Vector Operations Let $u, v, w \in \mathbb{R}^n$, $a, b \in \mathbb{R}$. Then

1. $u+v = v+u$
2. $(u+v)+w = u+(v+w)$
3. The zero vector $0 = \langle 0, \dots, 0 \rangle$ satisfies $0+v = v = v+0$
4. $v + (-1)v = 0 = (-1)v + v$. We write $-v = (-1)v$ and call $-v$ the additive inverse of v.
5. $(a+b)v = av + bv$
6. $a(u+v) = au + av$
7. $(ab)v = a(bv)$
8. $1 \cdot v = v$

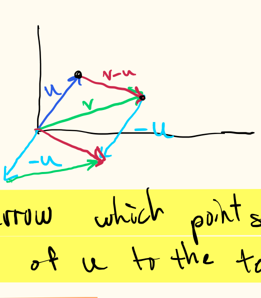
Section 9.2.5 Geometric Interpretation of Vector Operations

Let $u = \langle a,b \rangle$ and $v = \langle c,d \rangle$. Let $k \in \mathbb{R}$

Addition $u+v = \langle a+c, b+d \rangle$

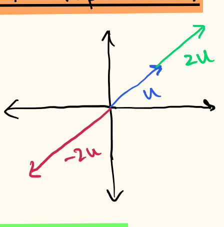


Subtraction $v - u = v + (-1)u$



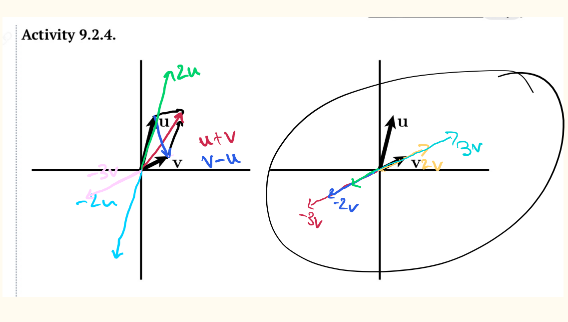
v-u is the arrow which points from the terminal point of u to the terminal point of v.

Scalar Multiplication



Activity 9.2.4

- Complete Activity 9.2.4 and discuss with your group.
- Class discussion.



(b) $0v = \langle 0,0 \rangle$
(d) All terminal points of vectors of the form tv lie on the line through the origin which is parallel to v.